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Effects of Octupole Vibrations on Quasiparticle Modes of Excitation in Superdeformed ^{193}Hg

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ABSTRACT

A particle-vibration coupling calculation based on the RPA and the cranked shell model has been carried out for superdeformed rotational bands in ^{193}Hg . The result suggests that properties of single-particle motions in superdeformed nuclei may be significantly affected by coupling effects with low-frequency octupole vibrational modes, especially by the lowest $K = 2$ octupole mode.

Since the shell structure of superdeformed nuclei is drastically different from that of ordinary deformed nuclei, we expect that new kinds of nuclear surface vibrational mode emerge above the superdeformed yrast states. In fact, the RPA calculation in the uniformly rotating frame, with the use of the single-particle states obtained by the cranked Nilsson-Strutinsky-BCS procedure, has indicated that we can expect highly collective, low-frequency octupole vibrational modes (with $K = 0, 1, 2$ and 3) about the superdeformed equilibrium shape.^{1,2)} Importance of the octupole correlations in superdeformed high-spin states has been discussed also in Ref. 3)~13). The main reason why the octupole is more favorable than the quadrupole is that each major shell consists of about equal numbers of positive- and negative-parity single-particle levels which are approximately degenerate in energy at the superdeformed shape.

Existence of low-frequency octupole modes would imply that particle-hole or quasi-particle modes of motion in superdeformed nuclei might be significantly affected by the coupling effects with these vibrational modes. In this paper, we report some results of theoretical calculation which indicate the importance of such particle-vibration coupling effects to understand the properties of Landau-Zener band-crossing phenomena recently observed in ^{193}Hg .¹⁴⁾

We solve the RPA equations for the Hamiltonian

$$H = h' - \frac{1}{2} \sum_K \chi_{3K} Q_{3K}^{\prime\prime\dagger} Q_{3K}^{\prime\prime} , \quad (1)$$

where h' is a cranked single-particle Hamiltonian of the Nilsson-plus-BCS type,

$$h' = h_{\text{Nilsson}} - \Delta \sum_i (c_i^\dagger c_i^\dagger + c_i c_i) - \lambda \hat{N} - \omega_{\text{rot}} \hat{J}_x , \quad (2)$$

and $Q_{3K}^{\prime\prime} = (r^3 Y_{3K})^{\prime\prime}$ are the doubly-stretched octupole operators.¹⁵⁾ We determine the equilibrium quadrupole deformation by means of the Strutinsky method and use a large configuration space composed of 9 major shells (for both protons and neutrons) when solving the coupled RPA dispersion equations. The octupole-force strengths χ_{3K} can be determined by the selfconsistency condition between the density distribution and the single-particle potential for the case of harmonic-oscillator potential.¹⁵⁾ However, the problem how to generalize this method to a more general single-particle potential like Eq. (2)

is not solved. Therefore, in this paper, we put $\chi_{3K} = f\chi_{3K}^{\text{HO}}$, where χ_{3K}^{HO} are the theoretical values¹⁵⁾ for the harmonic-oscillator potential, and regard f as a phenomenological parameter as well as the pairing gap Δ .

Figure 1 shows an example of the octupole strengths calculated at $\omega_{\text{rot}} = 0$ for the superdeformed ^{192}Hg . We see that the collectivity is highest for the $K = 2$ octupole mode. Figure 2 represents how the octupole strength distribution changes at a finite value of the rotational frequency ω_{rot} . In this figure, we can clearly see the K -mixing effects due to the Coriolis force; for instance, considerable mixing among the $K = 0, 1$ and 2 components is seen for the RPA eigenmode with excitation energy $\hbar\omega \approx 1.0\text{MeV}$.

Starting from the microscopic Hamiltonian (1) and using the standard procedure,¹⁶⁾ we can derive the following effective Hamiltonian describing systems composed of quasiparticle a_μ^\dagger and octupole vibrations X_n^\dagger ,

$$\mathcal{H} = \sum_\mu E_\mu a_\mu^\dagger a_\mu + \sum_n \hbar\omega_n X_n^\dagger X_n + \sum_n \sum_{\mu\nu} f_n(\mu\nu) (X_n^\dagger + \tilde{X}_n) a_\mu^\dagger a_\nu . \quad (3)$$

and we diagonalize it within the subspace $\{a_\mu^\dagger |0\rangle, a_\nu^\dagger X_n^\dagger |0\rangle\}$. The resulting state vectors can be written as

$$|\phi\rangle = \sum_\mu C_0(\mu) a_\mu^\dagger |0\rangle + \sum_n \sum_\nu C_1(\nu n) a_\nu^\dagger X_n^\dagger |0\rangle . \quad (4)$$

Recently, experimental data suggesting octupole correlations in superdeformed states have been reported by Cullen et al.¹⁴⁾ for ^{193}Hg . Figure 3 shows a result of calculation for excitation spectra in the rotating frame of this nucleus. By comparing the conventional quasiparticle energy diagram (Fig. 3-a)) with the result of diagonalization of \mathcal{H} (Fig. 3-b)), we can clearly identify effects of the octupole vibrations: Energy shifts $\Delta e'_{\text{vib}}$ of 50~300keV due to the coupling effects are seen. In particular, we note that the Landau-Zener crossing frequency ω_{cross} between band 1 (whose main component is the [512]5/2 quasiparticle state) and band 4 (associated with the [761]3/2 quasiparticle) is considerably delayed. Namely, we obtain $\omega_{\text{cross}} \approx 0.26\text{MeV}/\hbar$ in agreement with the experimental value¹⁴⁾ $\omega_{\text{rot}}^{\text{exp}} \approx 0.27\text{MeV}$, whereas $\omega_{\text{cross}} \approx 0.17\text{MeV}/\hbar$ if the octupole-vibrational effects are neglected. The reason for this delay is understood by examining the properties of the quasiparticle-vibration couplings in ^{193}Hg , which will be done below.

The amplitude $C_0(\mu)$, $C_1(\nu n)$ obtained by diagonalizing the effective Hamiltonian \mathcal{H} are displayed in Fig. 4 as functions of the rotational frequency ω_{rot} . Note that the K -quantum numbers used in this figure to label these amplitudes are valid only in the limit $\omega_{\text{rot}} \rightarrow 0$, because the K -mixing effects due to the Coriolis force are taken into account. It is seen that the main components of bands 1 and 4 exchange with each other at $\omega_{\text{rot}} \approx 0.26\text{MeV}/\hbar$ indicating the Landau-Zener crossing phenomena between the [512]5/2 and the [761]3/2 quasiparticle states. In this figure, we also see that the mixing of the states composed of the [624]9/2 quasiparticle and the $K = 2$ octupole vibration is significant in band 1. Note that there are two such states; $\left| [624]9/2(\alpha = -1/2) \otimes \omega_{K=2}^{(+)} \right\rangle$ and $\left| [624]9/2(\alpha = 1/2) \otimes \omega_{K=2}^{(-)} \right\rangle$ where α denotes the signature quantum number and $\omega_{K=2}^{(+)}$ and $\omega_{K=2}^{(-)}$ the octupole vibrations with positive and negative signatures, respectively, which reduce to the $K = 2$ octupole vibration shown in Fig. 1 in the limit of $\omega_{\text{rot}} = 0$.

It is worth emphasizing that $K = 2$ octupole matrix element between the [512]5/2 and the [624]9/2 Nilsson states is especially large since it satisfies one of the asymptotic selection rule ($\Delta N_{\text{sh}} = 1$, $\Delta n_3 = 1$, $\Delta \Lambda = 2$) for the transitions associated with the $K = 2$ octupole operator. (N_{sh} denotes the shell quantum number, defined by $N_{\text{sh}} = 2(n_1 + n_2) + n_3$.²⁾) Thus, these two Nilsson states strongly couple with each other due to the $K = 2$ octupole correlation. This property is seen also in the single-neutron energy diagram plotted as a function of the $K = 2$ octupole deformation parameter β_{32} in the paper by Skalski.¹⁰⁾ As a result of this property, we obtain a significant energy shift $\Delta e'_{\text{vib}}$ for band 1. On the other hand, the octupole vibrational effect is rather weak for band 4. Thus, as shown in Fig. 3, the relative excitation energy between bands 1 and 4 increases, so that their crossing frequency also increases. Furthermore, we can expect that this octupole correlation between bands 1 and 2 may contribute to the strong ($E1$) transitions from band 1 ([512]5/2) to band 2 ([624]9/2), which have been also reported in the experiment.¹⁴⁾

Next, let us discuss on the alignment i of band 4 and on the interaction matrix element V_{int} between bands 1 and 4, for which experimental data are available; $i_{\text{band}4}^{\text{exp}} \approx 1.3\hbar$ and $V_{\text{int}}^{\text{exp}} \approx 26\text{keV}$.¹⁴⁾ We evaluate the alignment by $i = -\partial E'/\partial \omega_{\text{rot}}$ using the eigenvalue E' of the effective Hamiltonian (3) and choosing the region of ω_{rot} where E' linearly depends on ω_{rot} . The interaction V_{int} is evaluated, as usual, from the half of the shortest distance between the energy levels for bands 1 and 4 in the energy diagram like Fig. 3-b). The calculated value of the alignment for the [761]3/2 quasiparticle state (the main component of band 4) is $i^{\text{cal}} \approx 1.8\hbar$. This value is reduced to $i^{\text{cal}} \approx 1.2\hbar$ in good agreement with

experiment, when the octupole-vibrational effects are taken into account. On the other hand, the interaction matrix element between the [761]3/2 quasiparticle state and the [512]5/2 quasiparticle (the main component of band 1) is almost zero and increases to about 5keV due to the octupole-vibrational effects. This calculated value of V_{int} is, however, too small in comparison with the experimental data.

Since we treat the doubly-stretched octupole force-strengths χ_{3K} as phenomenological parameters in this paper, it is necessary to examine the dependence on the force-strengths χ_{3K} , of the theoretical values for the crossing frequency ω_{cross} , the alignment $i_{\text{band}4}$ and the interaction matrix element V_{int} . This is done in Fig. 5. In this figure, the calculated values of ω_{cross} , $i_{\text{band}4}$ and V_{int} are plotted as functions of the excitation energy $\hbar\omega_{K=2}^{(-)}$ of the lowest $K = 2$ octupole vibration calculated at $\omega_{\text{rot}} = 0.45\text{MeV}/\hbar$, instead of plotting directly as functions of χ_{3K} . We note that $\hbar\omega_{K=2}^{(-)}$ is a function of χ_{3K} and the force-strengths $\chi_{3K} = 1.08\chi_{3K}^{\text{HO}}$ adopted in the calculations of Figs. 1~4 correspond to the abscissa at $\hbar\omega_{K=2}^{(-)} \approx 0.5\text{MeV}$ in Fig. 5. It is seen from this figure that ω_{cross} increases while $i_{\text{band}4}$ decreases when $\hbar\omega_{K=2}^{(-)}$ decreases (*i.e.*, when the octupole-vibrational effects become stronger), and we find that the experimental data for ω_{cross} and $i_{\text{band}4}$ are simultaneously reproduced at $\hbar\omega_{K=2}^{(-)} \approx 0.5\text{MeV}$. On the other hand, the calculated interaction matrix element V_{int} is too small within a reasonable range of $\hbar\omega_{K=2}^{(-)}$.

The main reason why the calculated value of V_{int} is small may be understood as follows: Generally speaking, we can expect that the band-band interactions increase due to the octupole-vibrational effects, because interactions between different quasiparticle states through intermediate configurations composed of one-quasiparticle and octupole vibrations become possible. However, in the specific case of band 4, as seen in Fig. 4, the octupole vibrational effects are rather weak because the octupole matrix element between the [761]3/2 Nilsson state and the neighboring Nilsson states are small. On the other hand, the octupole-vibrational effects are indeed strong in band 1 so that the considerable mixing of the states $|[624]9/2(\alpha = \mp 1/2) \otimes \omega_{K=2}^{(\pm)}\rangle$ occurs. This mixing does not, however, lead to a large interaction V_{int} between bands 1 and 4, because the octupole matrix element between the [624]9/2 and the [761]3/2 quasiparticle states remains small although the Coriolis K -mixing effects are taken into account in our calculation at finite rotational frequency.

The interaction V_{int} under consideration depends also on the pairing-gap parameter Δ as well as the force-strengths χ_{3K} . We have adopted $\Delta = 0.7\text{MeV}$ in Fig. 5 (cf. we

obtain $\Delta_p = 0.72\text{MeV}$ and $\Delta_n = 0.77\text{MeV}$ when the pairing gap is evaluated at $\omega_{\text{rot}} = 0$ by means of the conventional procedure of the Strutinsky method¹⁷⁾ with the pairing-force strengths G that gives the standard value of the smoothed pairing-gap parameter $\tilde{\Delta} = 12.0A^{-1/2}\text{MeV}$). The result of calculation using $\Delta = 0.9\text{MeV}$ was reported in Ref. 18). In this case, we obtain $V_{\text{int}} \approx 10\text{keV}$ keeping the agreement of ω_{cross} and i_{band4} with experiment. This value of V_{int} is still too small compared with the experimental value $V_{\text{int}} \approx 26\text{keV}$. Thus, we conclude that the large value of V_{int} cannot be reproduced within the present framework of calculation by merely changing the pairing-gap parameter Δ within a reasonable range.

In summary, we have investigated the coupling effects between the quasiparticle and the octupole-vibrational modes of excitation in the superdeformed ^{193}Hg , by means of the particle-vibration coupling theory based on the cranking model. We have found that the inclusion of the octupole vibrational effects is important to reproduce the experimental data for the crossing frequency between bands 1 and 4, and for the aligned angular momentum of band 4. On the other hand, the calculated interaction matrix element between bands 1 and 4 is too small in comparison with the experimental data. To understand the spectrum of the superdeformed ^{193}Hg , there are several problems remaining for the future; e.g., improvement of the treatment of the pairing correlations, inclusion of the quadrupole-pairing, evaluation of the $E1$ transition probabilities, possibilities of other interpretation of the experimental data,¹⁹⁾ etc.

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FIGURE CAPTIONS

- 1) Octupole strengths $|\langle n | (r^3 Y_{3K})'' | 0 \rangle|^2$ calculated for the superdeformed states of ^{193}Hg at $\omega_{\text{rot}} = 0$. Note that positive- and negative-signature states are completely degenerate at $\omega_{\text{rot}} = 0$ (for peaks with $K = 1, 2$ and 3). The deformation parameter $\delta_{\text{osc}} = 0.43$, the neutron gap $\Delta_n = 0.7\text{MeV}$, the proton gap $\Delta_p = 0.7\text{MeV}$ and the doubly-stretched octupole interaction strengths $\chi_{3K} = 1.08\chi_{3K}^{\text{HO}}$, χ_{3K}^{HO} being the selfconsistent values for the harmonic-oscillator potential, are used. The numbers written beside the main peaks indicate the strengths for the $E3$ operators measured in Weisskopf units.
- 2) The same as Fig. 1 but for $\omega_{\text{rot}} = 0.25\text{MeV}/\hbar$.
- 3) a) Quasiparticle energy diagram for neutrons with signature $\alpha = -1/2$ in ^{193}Hg , plotted as a function of ω_{rot} .
b) The same as a) but the energy shifts $\Delta e'_{\text{vib}}$ due to the coupling effects with the octupole vibrations are included. Parameters used in the calculation are the same as in Figs. 1 and 2. Notation like $[512]5/2$ indicate the main components of the wave functions.
- 4) Amplitudes $C_0(\mu)$ and $C_1(\nu n)$ in the wave function defined by Eq. (4), plotted as functions of ω_{rot} . The full lines are used for the one-quasiparticle amplitudes, while the broken (dotted) lines for the amplitudes involving the octupole vibrations with positive (negative) signature. (a), (b), (c) and (d) respectively show the results of calculation for bands 1, 2, $2'$ and 4. The main component of band 2 is the $[624]9/2(\alpha = 1/2)$ quasiparticle state. The observed band 2 was suggested in Ref. 14) that it could actually be two bands with identical γ -ray energies consisting of the $[624]9/2(\alpha = 1/2)$ band and the $[512]5/2(\alpha = 1/2)$ band. The latter band, which is the signature partner of band 1, is denoted here band $2'$. The parameter used in the calculation are the same as in Fig. 1.
- 5) Dependence of crossing frequency ω_{cross} between bands 1 and 4, the aligned angular momentum of band 4 $i_{\text{band}4}$, and the interaction matrix element V_{int} between bands 1 and 4, on the excitation energy $\hbar\omega_{K=2}^{(-)}$ of the lowest $K = 2$ octupole vibration (with negative signature) calculated at $\omega_{\text{rot}} = 0.45\text{MeV}$. The pairing gaps used are the same as in Fig. 1. The excitation energy $\hbar\omega_{K=2}^{(-)} = 0.54\text{MeV}$ corresponds to the force-strengths $\chi_{3K} = 1.08\chi_{3K}^{\text{HO}}$.